

## Thermodynamic Optimization of the Operative Parameters for the Heat Recovery in Combined Power Plants\*

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### Abstract

For the combined power plants, the optimization of the heat recovery steam generator (HRSG) is of particular interest in order to improve the efficiency of the heat recovery from turbine exhaust gas and to maximize the power production in the steam cycle. The thermodynamic optimization is the first step of a power plant design optimization process. The aim of this paper is to provide thermodynamic tools for the optimal selection of the operative parameters of the HRSG, starting from which a detailed optimization of its design variables can be carried out. For the thermodynamic analysis, the selected objective is the minimization of thermal exergy losses, taking into account only the irreversibility due to the temperature difference between the hot and cold streams. Various HRSG configurations have been analyzed, from the simpler, a single evaporator to the common configuration of two-pressure steam generator with five different sections.

*Key words: heat recovery steam generator, combined plants, exergy analysis.*

### 1. Introduction

In the field of the combined plant analysis, thermodynamic optimization of the heat recovery steam generator (HRSG) is of particular interest in order to improve the efficiency of the heat recovery from gas turbine exhaust and to maximize the power generated in the vapor cycle. At the same time it represents a means to reduce the environmental impact and the pollutant emissions.

A detailed optimization of the HRSG depends on the surface geometry (finned tubes, plate and fins, etc.) and on the flow arrangement, but the first step is the optimization of the operational parameters with the corresponding saturation temperatures such as mass flow ratio, gas and liquid inlet temperatures, and pressure levels.

The practical design of the HRSG is usually based on the concepts of pinch point and

approach point that govern the gas and steam temperature profiles (Linhoff and Hindmarsh, 1983). The pinch point represents the difference between the gas temperature leaving the evaporator and the saturation temperature, while the approach-point temperature is the difference between the water temperature leaving the economizer and the saturation temperature. Pinch and approach points take into account both thermodynamic and economical points of view: their values are derived from practical experience (Ganapathy, 1994).

Regarding the HRSG optimum design, the aforesaid method can be used as an iterative tool to improve the selected operative parameters under specific conditions, but in this way the economic and thermodynamic aspects are mixed and it is difficult to understand the real weight of each of them on the optimization result.

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A different optimization approach for the HRSG operative parameters can be based on a thermodynamic or thermo-economic analysis by minimizing a suitable objective function with analytical or numerical mathematical methods (Bejan et al., 1996). In order to identify the correct weight of thermodynamic and economic elements in the aforesaid objective function, a necessary first step is the splitting of the thermodynamic and economic aspects. In this paper the optimization of the HRSG has been performed using a thermodynamic criterion, the minimization of exergy losses, taking into account only irreversibility due to the temperature difference between the hot and the cold streams. Although this criterion gives solutions with zero pinch point temperature difference and infinite heat exchange surface, it allows to select a first solution of the operational parameters like mass flow ratio, gas and liquid inlet temperature and saturation temperatures.

## 2. Thermodynamic Optimization

Traditionally, the basis for the study of energy conversion systems has been the conservation of mass and the First Law of Thermodynamics. But efficiency based on the First law of Thermodynamics cannot properly assess performance of an HRSG when the final aim is the production of work (Wall and Gong, 1996; Gaggioli et al., 1978). For example, if a high mass flow rate of liquid water is used to recover energy from exhaust gases, the increase of the liquid temperature cannot be used in order to produce work even if the heat recovery effectiveness is unitary.

The use of the Second Law of Thermodynamics offers a more appropriate approach carrying out the thermodynamic analysis with the minimization of the irreversibility. The entropy generation minimization was first applied in heat exchangers by Bejan (1977), Ciampi and Tuoni (1979) and also recently by El Sayed (1996\*). But for a rational analysis of the heat recovery from the turbine exhaust gases, it is not sufficient to minimize the entropy generation in the heat transfer process, but it is also important to estimate the power generated in the vapor cycle. A possible means to this aim is presented by an exergy analysis of the HRSG performing the minimization of the exergy losses, by Cornelissen and Hirs (1999).

The exergy method is largely discussed in the classical books (Bejan et al., 1996; Kotas, 1995). A complete exergo economic analysis of a component like the HRSG would take into account both the physical environment (mass, energy, entropy, chemical potentials) and the

economic environment (manufacturing and operative costs, taxes, interest rates) in order to develop a thermo-economic optimization. But the problems appear to be very complex, both considering the single-phase heat exchanger (Aceves-Saborio et al., 1989; Cornelissen, 1998) and the two-phase one (Zubair et al., 1987). Considering a thermodynamic analysis of the problem, we will focus our attention on the thermal exergy losses neglecting the exergy losses due to the pressure drop, taking into account only the irreversibility due to the temperature difference between the hot and cold streams. The exergy losses due to the pressure drop can be considered in an expanded analysis of the HRSG. Their evaluation requires a detailed description of the HRSG, not available at the level of optimization assumed in the paper.

## 3. The Gas-Side Effectiveness (Single Phase Channel Effectiveness)

Each section of the HRSG is a heat exchanger for which the stream with minimum flow thermal capacity is not generally known. Therefore the classical heat exchanger effectiveness (Kays and London, 1984) cannot be directly applied to analyze and optimize the HRSG sections. The single-phase channel effectiveness has to be considered a generalization of the classical heat exchanger effectiveness, useful independently of the knowledge of the minimum flow thermal capacity, and in the present paper its use is proposed for the HRSG analysis, instead of the usual pinch-point method. In particular the above-mentioned quantity is the gas side effectiveness, also defined as "temperature effectiveness" (Kays and London, 1984) that for the single section of the HRSG is defined as:

$$\eta = \frac{T_{gin} - T_{gout}}{T_{gin} - T_{lin}} \quad (\text{single-phase channel}) \quad (1)$$

where  $T_{gin}$  and  $T_{lin}$  are the inlet gas and cold fluid temperatures, respectively and  $T_{gout}$  is the gas outlet temperature. If  $\rho = g/G$  is the ratio between the flow thermal capacity of cold fluid (g) and of the gas (G), the relation between the gas side effectiveness  $\eta$  and the classical one  $\varepsilon$  (Kays and London, 1984) is given by:

$$\eta = \varepsilon \quad \text{if } \rho \geq 1 \quad \text{or} \quad \eta = \rho\varepsilon \quad \text{if } \rho < 1 \quad (2)$$

so that the evaporator effectiveness is coincident with the classical one, while for the economizer and super-heater sections it represents the ratio of the gas temperature drop and the maximum temperature difference between gas and liquid. So the HRSG can be analyzed and optimized

only following the evolution of the gas that does not change phase. This is important because in the HRSG sections the thermal capacity ratio of the cold and hot streams is different, changing from one to the other section. The HRSG can be analyzed and optimized only following the evolution of the gas that would not change phase and can be simply extended to consider multi-pressure levels. In the single section of the HRSG the heat flow rate can be expressed in terms of the gas-side effectiveness while this last can be obtained starting from the classical efficiency depending on the flow arrangement. In the case of counter-flow arrangement, for economizer or super-heater, it is simple to obtain

$$\eta = \frac{1 - e^{-N(1-1/\rho)}}{1 - (1/\rho) \cdot e^{-N(1-1/\rho)}} \quad N = \frac{UA}{G} \quad (3)$$

where the variable  $\rho$  can assume all the real strictly positive values.

The trend of the gas-side effectiveness described by Eq. (3) is presented in *Figure 1* for particular values of  $\rho$  and  $N$  as parameters. If  $\rho > 1$ , the quantity  $N$  is coincident with the usual number of transfer units, but generally, for given values of the overall heat transfer coefficient  $U$  and of the gas thermal capacity  $G$ , it represents a measure of the heat transfer surface area  $A$ .

### 3. Thermodynamic Optimization of a Counter-Flow Heat Exchanger

Before starting the analysis of typical HRSG configurations, let us consider as demonstrative application the case of a single-phase counterflow heat exchanger where a gas with flow thermal capacity  $G$  is the hot fluid and a liquid with flow thermal capacity  $g$  is the cold fluid. The exergy destruction in the heat exchanger can be calculated as the difference between all incoming and outgoing exergy flows. Assuming the inlet temperature of cold fluid equal to the environmental temperature

( $T_{lin} = T_a$ ) and remembering the assumption of neglecting the exergy destruction due to the pressure drop, the exergy loss can be simply expressed in terms of the absolute temperatures of the fluids as:

$$\Delta Ex = I = G(T_{gout} - T_a) - t_a G \ln\left(\frac{T_{gin}}{T_a}\right) + t_a g \ln\left(\frac{T_{lout}}{T_a}\right) \quad (4)$$

The previous expression, elaborated in terms of the single-phase effectiveness of the hot stream, is a function of the efficiency, of  $\rho$  and of the parameter  $\theta = T_a / T_{gin}$

$$I^* = I / Gt_a = \ln(\theta) + \rho \cdot \ln\left(1 + \frac{\eta}{\rho} \cdot \frac{1-\theta}{\theta}\right) - 1 + \frac{1}{\theta} - \eta \cdot \frac{1-\theta}{\theta} \quad (5)$$

The function  $I^*$  can be represented graphically for different values of the parameter  $\theta$  (see *Figure 2*). It can be easily shown that it is possible to find a minimum of the exergy destruction. The minimum can be obtained for a value of the ratio between the two flow thermal capacities that is close to unity as the number of transfer units  $N$  defined in (3) increases.

### 4. Application of the Exergy Method to the Analysis of the Heat Recovery

After the consideration of the single-phase heat exchanger, let us now extend the optimization method, and the related theoretical considerations about the exergy destruction, to the two-phase heat recovery, i.e. to steam generators. We will examine in the following paragraphs various possible configurations, starting from the simpler one, represented by the single evaporator, to a more complex one represented by a usual two-pressure steam generator with five different sections.

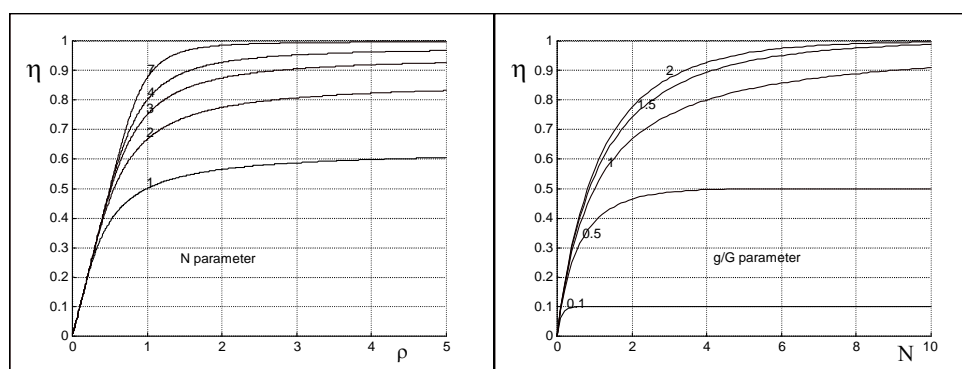


Figure 1. Gas-side effectiveness

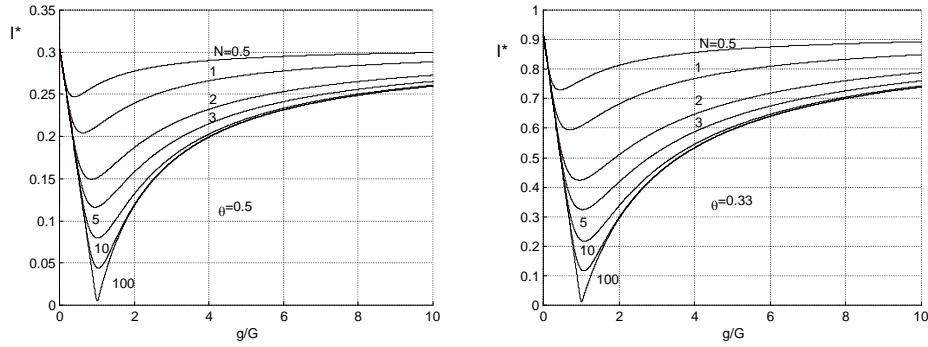


Figure 2. Exergy losses for single phase heat exchanger

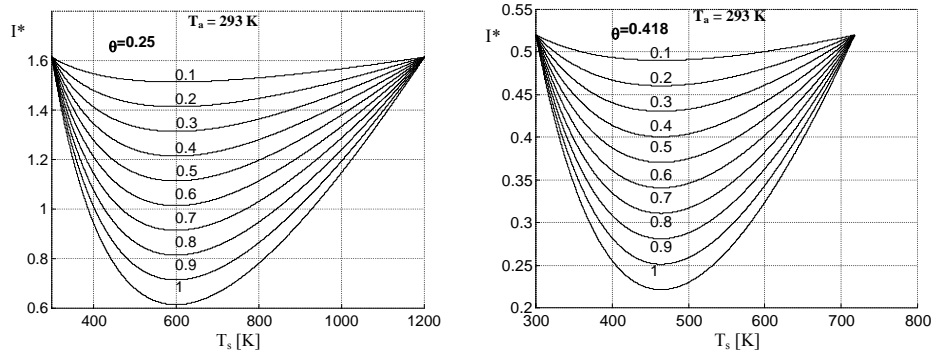


Figure 3. Exergy losses for single evaporator with  $\eta_v$  as parameter ( $\eta_v = 0.1-1$ )

#### 4.1 Heat recovery with evaporator only (Carnot cycle)

Applying the analysis to the single or multi-pressure level evaporator, a simple criterion to select the saturation temperature  $T_s$  as a function of inlet gas temperature  $T_{gin}$  and environmental temperature  $T_a$  can be given. By means of the evaporator effectiveness  $\eta_v$  (i.e. the gas-side effectiveness defined by Eq. (1) with  $T_{lin} = T_s$ ) and defining the parameters  $\theta = T_a / T_{gin}$  and  $\beta = T_s / T_{gin}$ , the exergy losses for a single pressure evaporator can be expressed in dimensionless form as:

$$I^* = \frac{I}{G \cdot t_a} = \left[ \frac{1-\theta}{\theta} + \ln(\theta) \right] - \eta_v \cdot \frac{(1-\beta)}{\theta} \cdot \left( 1 - \frac{\theta}{\beta} \right) \quad (6)$$

Minimizing the previous expression with reference to  $\beta$ , the result  $\beta_{opt} = \sqrt{\theta}$  and  $\eta_v = 1$  (i.e. null pinch point) can be obtained and consequently the optimum of the saturation temperature is given by:

$$T_{s,opt} = T_{gin} \cdot \sqrt{T_a / T_{gin}} \quad (7)$$

which can also be considered as the optimal temperature of the upper heat source of a Carnot cycle used for power recovery. Obviously, returning to the dimensional analysis, the saturation temperature strongly depends on the ratio between the environmental temperature and the inlet gas one. Two particular values of the inlet gas temperature are considered in Figure 3. The same considerations can be extended to multi-pressure evaporator, and the result defined by Eq. (7) can be easily generalized, obtaining for the  $i^{th}$  pressure level an unitary evaporator effectiveness (i.e. null pinch point) and a saturation temperature  $T_{si}$  given by:

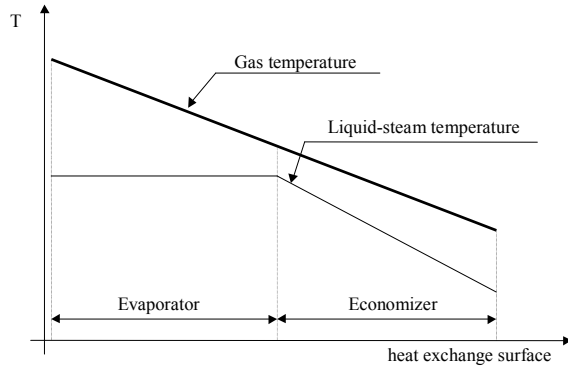
$$T_{si} = T_{gin} (T_a / T_{gin})^{i/(n+1)} \quad (8)$$

with  $n$  number of pressure levels. The results of Eqs. (7) and (8), obtained in ideal conditions, represent a first thermodynamic design criterion for the HRSG, even if in the real application the optimal saturation temperatures will be different from the “Carnot values” given by (8).

#### 4.2 Heat recovery with Rankine cycle

The thermodynamic optimization can be applied to the HRSG with economizer and

evaporator (Rankine cycle) and temperature profile sketched in *Figure 4*.



*Figure 4. Sketch of HRSG temperature profile for Rankine cycle*

According to the previously exposed theory, two gas-side effectiveness can be introduced: one for the economizer  $\eta_E$  and the other for the evaporator  $\eta_V$ . Defining the dimensionless parameter  $\gamma = T_{lin} / T_{gin}$  that must be joined with the previously defined  $\theta$ ,  $\beta$  and  $\rho$  as ratio of the flow thermal capacity of liquid and gas in the economizer, the exergy losses can be expressed as:

$$I^* = \frac{I}{Gt_a} = \frac{1-\theta}{\theta} + \ln \theta - \eta_V \frac{1-\beta}{\theta} \cdot \left(1 - \frac{\theta}{\beta}\right) - \rho \cdot \frac{\beta-\gamma}{\theta} \cdot \eta_E + \rho \ln \left(\frac{\beta}{\gamma}\right) \quad (9)$$

Applying the energy balance in the HRSG sections, it is possible to show that there exist the relations:

$$\eta_E = \eta_V \cdot \frac{\omega \cdot (1-\beta) \cdot (\beta-\gamma)}{1-\gamma-\eta_V \cdot (1-\beta)} \quad (10)$$

with  $\omega = \frac{c_l \cdot T_{gin}}{r}$

$$\rho = \omega \cdot \eta_V \cdot (1-\beta) \quad (11)$$

where  $c_l$  is the mean specific heat of the liquid in the temperature range  $T_{lin}$  to  $T_s$ . Eq. (9) gives the objective function  $I^*$ , which is to be minimized in the two variables  $\eta_V$  and  $\beta$ . It is quite easy to observe that the minimum of  $I^*$  is obtained for  $\eta_V = 1$  (zero pinch point temperature difference) and  $\eta_E = \rho$  (i.e.  $\varepsilon_E = 1$ ). With these values, examining the variation of the

exergy destruction with the variable  $\beta$ , it is easy to observe that there is a minimum that strongly depends on inlet gas temperature. This permits estimating the optimal value of the saturation temperature that results are, for the same conditions, lower than the one determined with the single evaporator. In the case of HRSG (water as cold fluid), given as input data:

$$T_{gin} = 700 \text{ K} \quad T_{lin} = 313 \text{ K} \quad T_a = 293 \text{ K}$$

so that  $\gamma = 0.447$   $\theta = 0.418$

the application of the thermodynamic optimization gives:

$$T_s = 490 \text{ K} \quad p_s = 21.9 \text{ bar}$$

( $c_l = 4.296 \text{ kJ/kg K}$ ;  $r = 1871.08 \text{ kJ/kg}$ ;  $\omega = 1.607$ )

$$\rho = 0.48 \quad \eta_V = 1 \quad \eta_E = 0.48$$

$$T_{gout} = 406 \text{ K} \quad I^*_{min} = 0.155$$

while for the single pressure evaporator, applying Eq. (7) it is possible to obtain:

$$\beta_{opt} \cong 0.647; \quad T_s \cong 452 \text{ K}; \quad p_s \cong 9.8 \text{ bar};$$

$$I^*_{min} \cong 0.22$$

The optimum value of the ratio of the saturation temperature to the inlet gas temperature depends strongly on  $T_{gin}$ . In *Figure 5* a comparison between two particular values of the inlet gas temperature is shown.

Regarding *Figure 5*, it is important to observe that the upper limit temperature of the saturation temperature is lower than the critical temperature of water (647.3 K), but it is particularly bounded by the minimum permitted value of the steam quality at the turbine outlet, corresponding to an assigned value of the steam turbine efficiency.

*Figure 5* shows that the curves describing the exergy losses, after a minimum, show an increase and then a rapid decrease in the proximity of the critical point (e.g. case  $T_{gin} = 700 \text{ K}$ ) related to the thermodynamic properties of water close to the critical point. It is remarkable to note that with the increase of the inlet gas temperature (e.g. case  $T_{gin} = 780 \text{ K}$ ), the relative minimum tends to disappear and the minimum exergy losses would be obtained with a supercritical cycle.

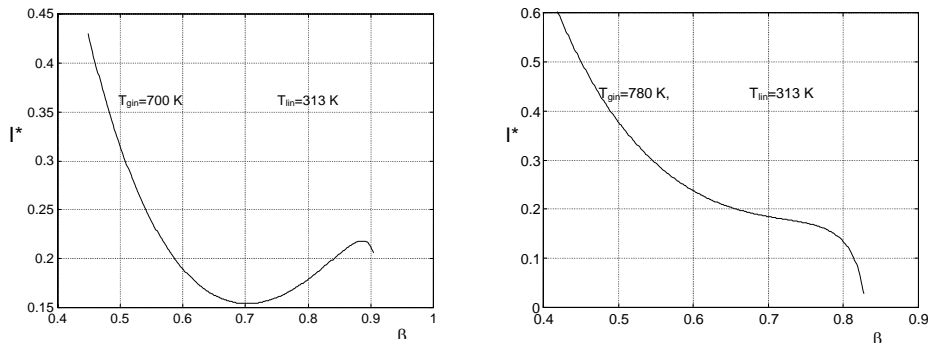


Figure 5. Exergy losses as a function of the saturation temperature  $t_s$

### 4.3 Heat recovery with Hirn cycle

The most general configuration of HRSG is the one with the superheater, too (Hirn cycle). In this case it is possible to define, in addition to the economizer and evaporator gas-side effectiveness, the super-heater gas-side effectiveness  $\eta_s$  (Figure 6).

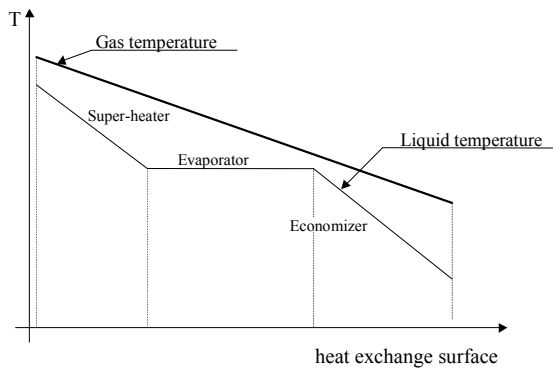


Figure 6. Sketch of HRSG temperature profile for Hirn cycle

For this configuration, let us define the additional parameter  $\psi = c_v / c_l$ , with  $c_v$  and  $c_l$  average specific heat of super-heated steam in the range  $T_s \div T_{lout}$  and of the liquid in the range  $T_{lin} \div T_s$ , respectively. Using all the defined dimensionless parameters, the exergy destruction related to the heat recovery is expressed by:

$$I^* = \frac{I}{Gt_a} = \frac{1-\theta}{\theta} + \ln \theta - \frac{1-\beta}{\theta} [\eta_s + \eta_v(1-\eta_s)] \cdot (1-\eta_E) - \frac{1-\gamma}{\theta} \eta_E - \rho \ln \left( \frac{\gamma}{\beta} \right) + \frac{1-\beta}{\beta} \eta_v(1-\eta_s) + \rho \psi \ln \left[ 1 + \frac{\eta_s}{\rho \psi} \frac{1-\beta}{\beta} \right] \quad (12)$$

The details about the derivation of Eq. (12), as well as of Eq. (9) and Eq. (6) that are only

particular cases, are given in Appendix 1. From the energy balance in the HRSG sections, it is possible to show that there exist the relations:

$$\rho = \eta_v \cdot \omega \cdot (1-\beta) \cdot (1-\eta_s) \quad (13)$$

$$\eta_s = 1 - \frac{\bar{\eta}_v}{\eta_v - \bar{\eta}_v} \cdot \frac{\beta - \gamma}{1 - \beta} \quad \text{with}$$

$$\bar{\eta}_v = \frac{\eta_E}{\eta_E + \omega \cdot (\beta - \gamma)} \quad (14)$$

Considering the relations (13) and (14), the exergy destruction (12) that has to be minimized is the function of the three independent variables  $\eta_E$ ,  $\eta_v$  and  $\beta$ .

In this case the study of the partial derivatives is very difficult so that a parametric study or a usual numerical minimization method is preferred (Rao, 1996). The minimum of the exergy destruction occurs for a null pinch point ( $\eta_v = 1$ ).

If  $T_{gin} = 700$  K and  $T_{lin} = 313$  K it is:

$$\begin{aligned} \beta_{opt} &\cong 0.63 & T_s &\cong 442 \text{ K} & p_s &\cong 8 \text{ bar} \\ I^*_{min} &\cong 0.125 & \eta_E &\cong 0.415 \\ \eta_v &\cong 1 & \eta_s &\cong 0.2 & \rho &= 0.415 \\ T_{lout} &\cong 670 \text{ K} & T_{gout} &\cong 387 \text{ K} \end{aligned}$$

The optimization results lead to a unitary efficiency of the economizer ( $\varepsilon_E \cong 1$ ), while for the super-heater it is  $\varepsilon_s = \eta_s / \rho \cong 0.48$ .

### 4.4 Heat recovery with double pressure level (Hirn and Rankine cycles)

The optimization method proposed in the paper can be extended to a Rankine+Hirn cycle configuration (double pressure level HRSG), that represents a more common case for the HRSG. In this case, following the general method exposed, it is possible to define five gas-side effectivenesses, two for the low pressure level, and three for the high pressure level. The exergy

destruction assumes a more complex structure. It is possible to demonstrate that it depends on five independent variables. Three variables are the gas-side effectivenesses,  $\eta_{VH}$ ,  $\eta_{EH}$ , of the evaporator and of the economizer at high pressure (Hirn cycle) and  $\eta_{VR}$ , of the evaporator at low pressure (Rankine cycle). The two remaining variables are the ratios between the saturation temperature and the inlet gas temperature, for the high pressure  $\beta_H$ , and for the low pressure  $\beta_R$ . Because the effectiveness of vaporization has to be assumed unitary both for the Rankine cycle and for the Hirn cycle, the only three variables of the minimization problem remain the two saturation temperatures and the gas-side effectiveness of high-pressure economizer. In *Figure 7* besides the characteristic HRSG temperature profile, a particular result of the optimization obtained by means of a numerical method, is reported.

The right side of *Figure 7* shows the inlet and outlet temperatures of each HRSG section for an input gas temperature  $T_{gin} = 700$  K; the temperature profile illustration is qualitative and relative to a very large heat exchange surface for the evaporators. The results obtained depend strongly on the input gas temperature and in TABLE I a sensitivity analysis applied to its variation is shown.

In this Table,  $T_{s1c}$  and  $T_{s2c}$  are the saturation temperatures given by Eq. (8),  $\rho_1$  and  $\rho_2$  the mass flow ratio in the high- and low-pressure sections,  $T'$ ; gas temperature at the inlet of the high-pressure evaporator. It is remarkable that if the inlet gas temperature is lower than 750 K, the saturation temperatures  $T_{s1}$  and  $T_{s2}$  are quite close to  $T_{s1c}$  and  $T_{s2c}$  obtained with two evaporators. Furthermore, extrapolating the results of TABLE I, it would be expected that for an inlet gas temperature higher than 800 K, the optimum high saturation temperature approaches the critical point of water (647.3 K) and the optimum recovery cycle tends to become a supercritical one. The results obtained by means of this analysis, though if only ideal determining infinite heat exchange surfaces and relative to simple cases, represent a method for having a first selection criterion for the operative parameters. The method can be simply extended to more complex HRSG configurations, like those with two or three pressure levels and reheat sections. In this case it will not be possible to obtain an analytical expression of the exergy losses, but numerical method will be necessary.

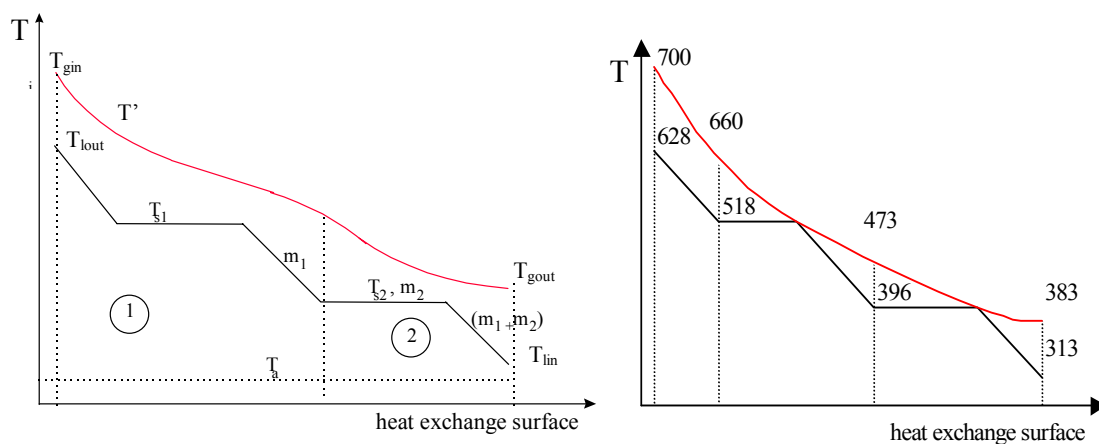


Figure 7. HRSG temperature profile for two pressure levels (Rankine+Hirn)

TABLE I. OPTIMIZATION RESULTS FOR A TWO-PRESSURE LEVELS HRSG

$T_{gin}$ [K]	$T_{gout1}$ [K]	$T_{s1}$ [K]	$T'$ [K]	$T_{lout}$ [K]	$T_{s2}$ [K]	$T_{gout}$ [K]	$\rho_1$	$\rho_2$	$I^*_{min}$	$T_{s1c}$ [K]	$T_{s2c}$ [K]
650	455	487	622	596	383	373	0.31	0.14	0.072	497	382
700	473	518	660	628	396	383	0.36	0.15	0.076	523	391
750	493	553	694	658	411	395	0.42	0.16	0.075	547	400
800	534	622	704	664	436	411	0.47	0.20	0.065	572	409

## 5. Conclusions

In the paper a thermodynamic optimization of the operative parameters for the heat recovery steam generator is carried out by means of the minimization of exergy losses. Taking into account only irreversibility due to the temperature difference between the hot and the cold streams, the analysis is based on the gas-side effectiveness of the sections of the HRSG instead of the usual pinch-point method.

Starting from this particular definition of effectiveness, the HRSG can be analyzed and optimized only following the evolution of the gas that does not change phase. Simple configurations like single pressure evaporator and a more realistic configuration like double pressure HRSG are considered. All the solutions derived by exergy analysis furnish null pinch-point and infinite surface. The results, even if meaningless from a technical point of view because of infinite surface and costs of HRSG, represent a first rough selection criterion for the HRSG operative parameters.

In order to find a compromise between high thermodynamic efficiency and low cost of the HRSG, a further development of the analysis will be thermo-economic optimization. A possible strategy is to minimize the objective function defined as the total cost of the HRSG. The costs related to exergy losses have to be recast to operative costs of the HRSG and combined with the installations costs.

Starting from the idea that a thermodynamic optimization is the first step of a plant optimization, the economical considerations will sensibly change the results obtained in the present paper with the minimization of exergy losses. Depending on the HRSG operative costs, the pinch-point temperature difference will assume values different from zero, but its value will be a well-defined function of the ratio between HRSG cost and the cost of the exergy losses. To assume a well definite pinch-point value means to have selected an exact value of the ratio between the two aforementioned costs.

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## Nomenclature

A	heat exchange surface (m <sup>2</sup> )
c	specific heat (J/kg K)
G	gas capacity rate (kJ/s)
g	liquid-steam capacity rate (kg/s)

Ex	exergy (J)
I	exergy losses (J)
I*	dimensionless exergy losses
N = UA/G	number of transfer units
P.P.	pinch-point (K)
p	pressure (bar)
r	latent heat of vaporization (J/kg)
T <sub>a</sub>	environment temperature (K)
T <sub>g</sub>	gas temperature (K)
T <sub>l</sub>	liquid (vapor) temperature (K)
T <sub>s</sub>	saturation temperature (K)
U	overall heat transfer coeff. (W/m <sup>2</sup> K)

## Greek symbols

ε	classical effectiveness
η	gas-side effectiveness
ρ	ratio between thermal capacity rate
θ	T <sub>a</sub> /T <sub>gin</sub>
γ	T <sub>lin</sub> /T <sub>gin</sub>
β	T <sub>s</sub> /T <sub>gin</sub>
ω	(c <sub>l</sub> T <sub>gin</sub> )/r
ψ	c <sub>v</sub> /c <sub>l</sub>

## Subscripts

a	environmental
c	for single evaporator (Carnot cycle)
E	relative to the economizer
g	of the gas
l	of the liquid
in	inlet
H	Hirn
min	minimum value
out	outlet
opt	optimal value
R	Rankine
s, sat	saturation
S	relative to the superheater
V	relative to the evaporator
v	of the vapor phase
1	relative to high pressure level
2	relative to low pressure level

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## APPENDIX 1: Analytical derivation of exergy losses equations

The expression of the exergy losses contained in Eqs. (6), (9) and (12) can be simply derived by applying the exergy balance to the particular HRSG configuration. Let us consider the single pressure HRSG with economizer and superheater (Hirn cycle configuration) whose temperature profiles are contained in *Figure A1* and let us derive the general expression of the exergy losses.

The three gas-side effectiveness or temperature effectiveness of the HRSG sections can be defined:

$$\eta_E = \frac{T'' - T_{gout}}{T'' - T_{lin}} \quad \text{economizer effectiveness} \quad (A.1)$$

$$\eta_V = \frac{T' - T''}{T' - T_s} \quad \text{evaporator effectiveness} \quad (A.2)$$

$$\eta_S = \frac{T_{gin} - T'}{T_{gin} - T_s} \quad \text{superheater effectiveness} \quad (A.3)$$

In this way, the unknown temperatures,  $T'$ ,  $T''$  and  $T_{gout}$  can be expressed as functions of the known temperature  $T_{lin}$ , of the saturation temperature  $T_s$ , as well as of the gas-side effectiveness of the three sections so that:

$$T' = T_{gin} - \eta_S \cdot (T_{gin} - T_s) \quad (A.4)$$

$$T'' = T_{gin} - [\eta_S + \eta_V \cdot (1 - \eta_S)] \cdot (1 - \eta_E) \cdot (T_{gin} - T_s) \quad (A.5)$$

$$T_{gout} = T_{gin} - \eta_E \cdot (T_{gin} - T_{lin}) - [\eta_S + \eta_V \cdot (1 - \eta_S)] \cdot (1 - \eta_E) \cdot (T_{lin} - T_s) \quad (A.6)$$

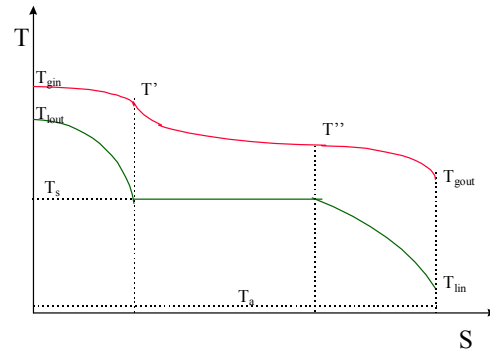


Figure A1. Sketch of HRSG temperature profile for Hirn cycle

The exergy loss  $I$  in the HRSG can be expressed in the general form:

$$I = Ex_{gin} + Ex_{lin} - Ex_{gout} - Ex_{lout} \quad (A.7)$$

or taking into account that  $Ex_{gout} = Ex_a$ , it is:

$$I = M \cdot (H_{gin} - T_a \cdot S_{gin}) + m \cdot (H_{gin} - T_a \cdot S_{lin}) - M \cdot (H_a - T_a \cdot S_a) - m \cdot (H_{gout} - T_a \cdot S_{lout}) \quad (A.8)$$

After short algebraic passages it is possible to write  $I$  only as a function of temperatures and:

$$I = M \cdot (H_{gout} - H_a) - McT_a \cdot \ln\left(\frac{T_{gin}}{T_a}\right) - mT_a \left[ -c_L \ln\left(\frac{T_s}{T_{lin}}\right) - \frac{r}{t_s} - c_v \ln\left(\frac{T_{gout}}{T_s}\right) \right] \quad (A.9)$$

$$I = G \cdot (T_{\text{gout}} - T_a) - GT_a \ln\left(\frac{T_{\text{gin}}}{T_a}\right) - T_a g \ln\left(\frac{T_s}{T_{\text{lin}}}\right) + \frac{T_a}{T_s} mr + g \frac{c_v}{c_L} T_a \ln\left(\frac{T_{\text{lout}}}{T_s}\right) \quad (\text{A.10})$$

Now, taking into account the previous relation, it is possible to express the temperature differences as a function of the known quantities and of the gas-side effectiveness and:

$$T_{\text{gout}} - T_a = T_{\text{gin}} - T_a - [\eta_s + \eta_v \cdot (1 - \eta_s)] \cdot (1 - \eta_E) \cdot (T_{\text{gin}} - T_s) - \eta_E (T_{\text{gin}} - t_{\text{lin}}) \quad (\text{A.11})$$

$$mr = G\eta_v (T' - T_s) = G\eta_v (1 - \eta_s) (T_{\text{gin}} - T_s) \quad (\text{A.12})$$

$$T_{\text{lout}} - T_s = \frac{G}{g} \frac{c_L}{c_v} (T_{\text{gin}} - T') = \frac{G}{g} \frac{c_L}{c_v} \eta_s (T_{\text{gin}} - T_s) \quad (\text{A.13})$$

so that the dimensionless exergy losses can be expressed in the form:

$$I^* = \frac{I}{Gt_a} = \frac{(T_{\text{gout}} - T_a)}{T_a} - \ln\left(\frac{T_{\text{gin}}}{T_a}\right) + \frac{g}{G} \ln\left(\frac{T_s}{T_{\text{lin}}}\right) + \eta_v (1 - \eta_s) \frac{T_{\text{lin}} - T_s}{T_s} + g \frac{c_v}{c_L} \ln\left(1 + \frac{G}{g} \frac{c_L}{c_v} \eta_s \frac{T_{\text{lin}} - T_s}{T_s}\right) \quad (\text{A.14})$$

and inserting the dimensionless quantities:

$$\rho = g / G \quad \theta = T_a / T_{\text{gin}} \quad \gamma = T_{\text{lin}} / T_{\text{gin}} \\ \beta = T_s / T_{\text{gin}} \quad \psi = c_v / c_L$$

the Eq. (12) can be obtained:

$$I^* = \frac{1 - \theta}{\theta} + \ln \theta - \frac{1 - \beta}{\theta} [\eta_s + \eta_v (1 - \eta_s)] \cdot (1 - \eta_E) - \frac{1 - \gamma}{\theta} \eta_E - \rho \ln\left(\frac{\gamma}{\beta}\right) + \frac{1 - \beta}{\beta} \eta_v (1 - \eta_s) + \rho \psi \ln\left[1 + \frac{\eta_s}{\rho \psi} \frac{1 - \beta}{\beta}\right] \quad (12)$$

If in Eq. (12) we suppose  $\eta_s=0$  the expression of exergy losses for the Rankine cycle, referred in the text as Eq. (9) can be simply derived:

$$I^* = \frac{I}{Gt_a} = \frac{1 - \theta}{\theta} + \ln \theta - \eta_v \frac{1 - \beta}{\theta} \cdot \left(1 - \frac{\theta}{\beta}\right) - \rho \cdot \frac{\beta - \gamma}{\theta} \cdot \eta_E + \rho \ln\left(\frac{\beta}{\gamma}\right) \quad (9)$$

If also  $\eta_E=0$ , so that the HRSG is a simple evaporator, adding the condition that  $\beta=\gamma$  the last term of Eq. (9) vanishes, it is simple to derive the expression of exergy losses for the Carnot cycle HRSG, referred in the text as Eq. (6), that is:

$$I^* = \frac{I}{G \cdot t_a} = \left[ \frac{1 - \theta}{\theta} + \ln \theta - \eta_v \cdot \frac{1}{\theta} \cdot (1 - \beta) \cdot \left(1 - \frac{\theta}{\beta}\right) \right] \quad (6)$$